* In the proof of Proposition 1.15, the term $\|R_\alpha\|_{H^s}$ should be replaced by the sum over $|\alpha| \leqslant s$ of $\|R_\alpha\|_{L^2}$, and in the proof of Proposition 2.2, the term $\|R_s^\varepsilon\|_{H^s}$ should be replaced by $\|R_s^\varepsilon\|_{L^2}$.
* p.31, bottom: "up to replacing V by $V + f(0)$ and f by $f - f(0)$" should be "up to replacing V by $V + \varepsilon^\alpha f(0)$ and f by $f - f(0)$"

In Chapter 2, the continuity argument is not used properly. Even though the results stated are essentially valid, they are not as precise as they should be. The needed modifications are listed below.
* p.32: the statement of Proposition 2.2 should be

Proposition. *Let Assumptions 1.7 and 2.1 be satisfied. There exists $T_0 \in (0, T]$, where T is given by Proposition 1.9, such that (2.2) has a unique solution $a^\varepsilon \in C([-T_0, T_0]; H^{s_0})$. Moreover, $(a^\varepsilon)_\varepsilon$ is bounded in $C([-T_0, T_0]; H^{s_0})$. If $(a_0^\varepsilon)_\varepsilon$ is bounded in H^s for some $s \geqslant s_0$, then $(a^\varepsilon)_\varepsilon$ is bounded in $C([-T_0, T_0]; H^s)$.*

This is indeed the result provided by the continuity argument on p.34. Consequently, T should be replaced by T_0 in Corollary 2.4.
* p.35: the statement of Proposition 2.5 is essentially correct, but should be

Proposition. *Let Assumptions 1.7 and 2.1 be satisfied, as well as (2.4). Then there exist $C > 0$ and $\varepsilon_0 \in (0, 1]$ such that*

$$\|a^\varepsilon - \widetilde{a}^\varepsilon\|_{L^\infty([-T,T];H^{s-2})} \leqslant C\left(\varepsilon + \|a_0^\varepsilon - a_0\|_{H^{s-2}}\right), \quad 0 < \varepsilon \leqslant \varepsilon_0.$$

The fact that T_0 can be extended to T stems from the analysis of Section 2.3 (which shows that $\widetilde{a}^\varepsilon$ remains smooth on $[-T, T]$), whose content should therefore be moved before Proposition 2.5. The assumption that ε should be sufficiently small stems from a bootstrap argument, since (2.4) and the error estimate show that for ε sufficiently small, $\|a^\varepsilon(t)\|_{L^\infty}$ remains bounded on $[-T, T]$.
* p.38: in Proposition 2.6, the assumption $0 < \varepsilon \leqslant \varepsilon_0$ should be added, for the exact solution a^ε need not be smooth up to time T for ε "large". In addition, a power of ε is missing in the error estimate, which should read

$$\left\|a^\varepsilon - a^{(0)} - \varepsilon a^{(1)}\right\|_{L^\infty([-T,T];H^{s-4})} \leqslant C\left(\varepsilon^2 + \|a_0^\varepsilon - a_0 - \varepsilon a_1\|_{H^{s-4}}\right).$$

In Chapter 3, some estimates on the modulated energy are not correct.
* p.55, the inequality $\theta(a) + \theta(b) \leqslant K\theta(a + b)$ is actually false. To overcome this issue, the following convexity lemma can be used:

Lemma. *There exists $K > 0$ independent of m such that for all $\rho', \rho \geqslant 0$,*

$$|G_m(\rho') - G_m(\rho) - (\rho' - \rho)G_m'(\rho)| \leqslant K\left|F_m(\rho') - F_m(\rho) - (\rho' - \rho)F_m'(\rho)\right|.$$

This lemma stems from Taylor formula with an integral remainder, and the identities

$$F_m''(y) = f_m'(y) \quad ; \quad G_m''(y) = f_m'(y) + y f_m''(y).$$

Setting, for $y \geqslant 0$, $h(y) = y^\sigma/(1 + y^\sigma)$ we have:

$$f_m'(y) = \delta_m^{1-\sigma} h'(\delta_m y) \quad ; \quad f_m''(y) = \delta_m^{2-\sigma} h''(\delta_m y).$$

Moreover,

$$h'(y) = \frac{\sigma y^{\sigma-1}}{(1 + y^\sigma)^2} \geqslant 0.$$

Therefore, to prove the lemma, it suffices to note that

$$yh''(y) = h'(y) \times \frac{\sigma - 1 - (\sigma + 1)y^\sigma}{1 + y^\sigma},$$

hence for all $y \geqslant 0$,

$$|yh''(y)| \leqslant Ch'(y).$$

The lemma implies

$$\frac{d}{dt} H_m^\varepsilon \leqslant C \left(H_m^\varepsilon + \varepsilon^2 \right) + o_{m \to \infty}(1),$$

for some C independent of m, and the conclusion follows like on p.55.

* p.70, a gradient is missing in the first displayed equation, which should be:

$$\partial_t \operatorname{Re} \left(\overline{a} a^{(1)} \right) + \nabla \Phi \cdot \nabla \operatorname{Re} \left(\overline{a} a^{(1)} \right) = -\frac{1}{2} \operatorname{div} \left(|a|^2 \nabla \Phi^{(1)} \right) - \operatorname{Re} \left(\overline{a} a^{(1)} \right) \Delta \Phi.$$